Closing Tues: 12.5(2), 12.5(3)
Closing Thur: 12.6, 13.1
Closing *next* Tues: 13.2, 13.3
Closing *next* Thur: 13.4

Exam 1 is next Thurs (April 19)
covers 12.1-12.5, 13.1-13.4

12.6: Intro to 3D surfaces

Goal: 7 basic 3D shapes and names

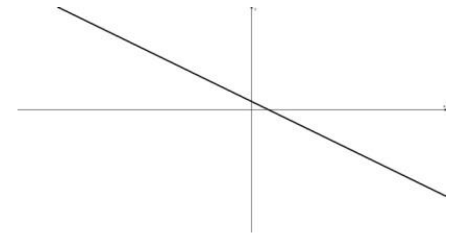
- Cylinders, Cones, Ellipsoids
- Paraboloids (two types)
- Hyperboloids (two types)

Entry Task: What are the names of these 2D curves?

1. $3x + 2y = 1$
2. $3x^2 - y = 4$
3. $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$
4. $\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

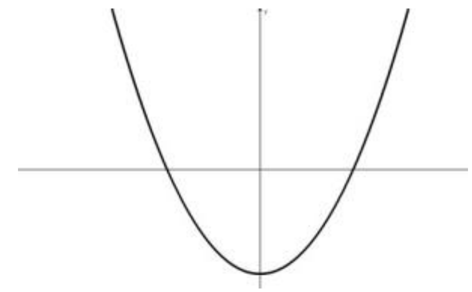
A 2D curve review

Lines: $ax + by = c$



Parabolas: $ax^2 + by = c$ or

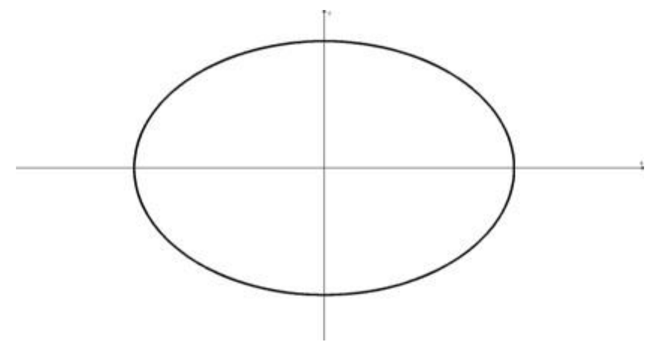
$ax + by^2 = c$



Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

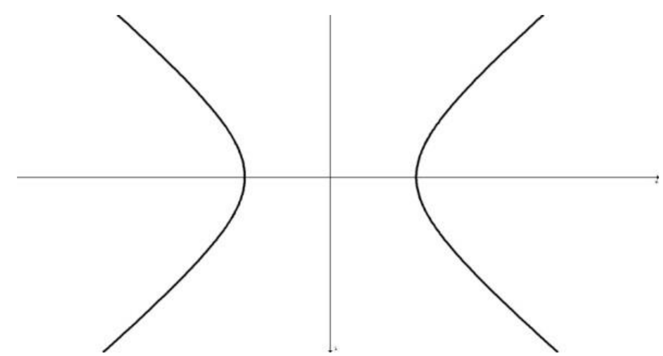
(*Note:* If a = b, then it's a circle)



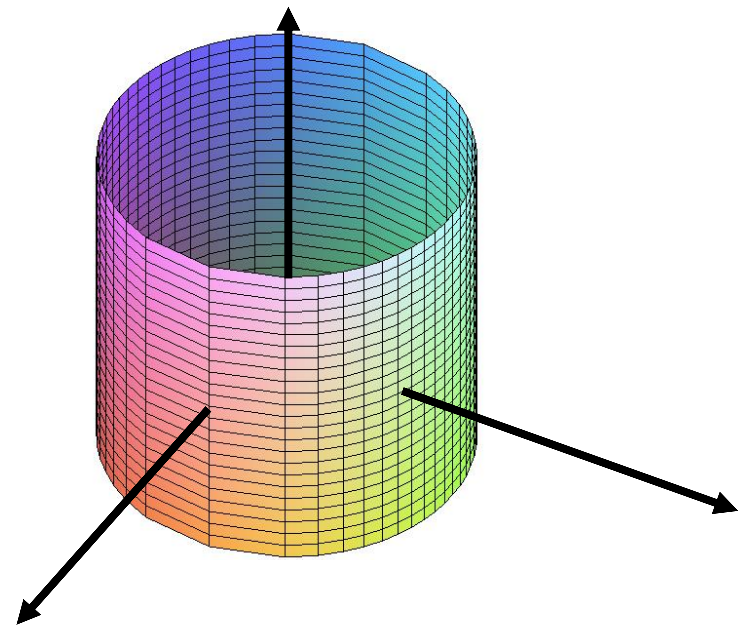
Hyperbola: $ax^2 - by^2 = c$ or

$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



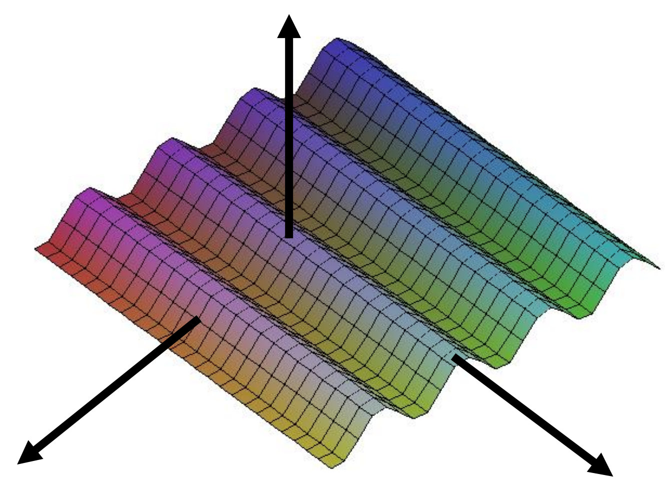
Cylinders: If *one variable is absent*, then the graph is a 2D curve extended into 3D.



If the 2D shade is called "BLAH", then the 3D shade is called a "BLAH cylinder".

Examples:

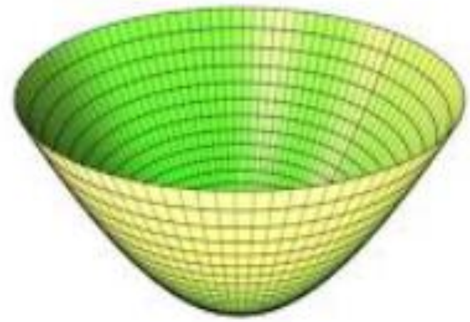
(a) $x^2 + y^2 = 1$ in 3D is a **circular cylinder** (*i.e.* a circle extended in the *z*-axis direction).



(b) $z = \cos(x)$ in 3D is a **cosine cylinder** (i.e. the cosine function extended in the *y*-axis direction).

Quadric Surfaces: A surface given by an equation involving a sum of first and second powers of x, y, and z is called a *quadric surface*.

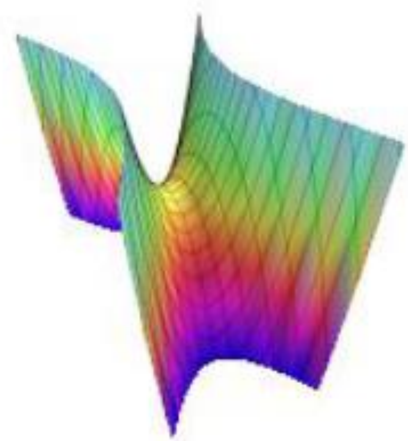
To visualize, we use **traces**.
We fix one variable and look at the resulting 2D picture (i.e. look at one vertical or horizontal slice). If we do several traces in different directions, we start to get an idea about the picture.



Elliptical/Circular Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

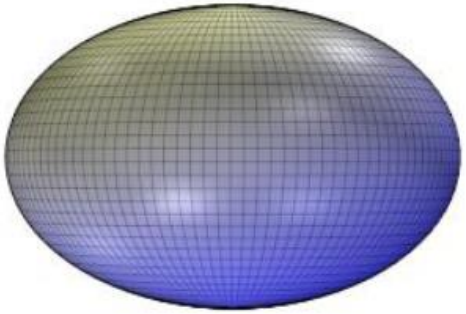
(*ex*: $z = 3x^2 + 5y^2$)



Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

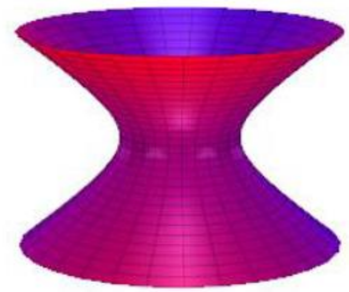
(*ex*: $y = 2x^2 - 5z^2$)



Ellipsoid/Sphere

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

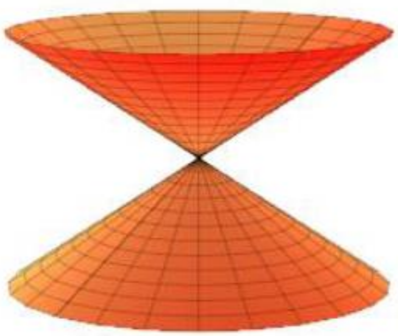
(*ex*: $3x^2 + 5y^2 + z^2 = 3$)



Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

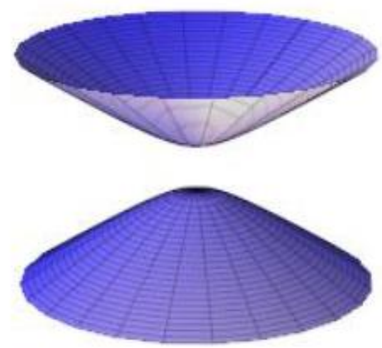
(*ex*: $x^2 - y^2 + z^2 = 10$)



Circular/Elliptical Cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

(*ex*: $z^2 = x^2 + y^2$)



Hyperboloid of Two Sheets

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

(*ex*: $x^2 + y^2 - z^2 = -4$)

Practice Examples

Find the traces and name the shapes:

1. $x - 3y^2 + 2z^2 = 0$
2. $4x^2+3y^2 = 10$
3. $5x^2 - y^2 - z^2 = 4$
4. $x^2 - 2y^2 + z^2 - 6 = 0$
5. $-x^2 + y^2 + 4z^2 = 0$

(Answers on the following pages)

1. $x - 3y^2 + 2z^2 = 0$

$x = k \rightarrow k - 3y^2 + 2z^2 = 0$ (hyperbola)
$y = k \rightarrow x - 3k^2 + 2z^2 = 0$ (**parabola**)
$z = k \rightarrow x - 3y^2 + 2k^2 = 0$ (**parabola**)
Also note: $x = 3y^2 - 2z^2$

Name: **Hyperbolic paraboloid**

2. $4x^2+3y^2 = 10$

One variable missing. Given equation is an ellipse.

Name: **Elliptical Cylinder**

3. $5x^2 - y^2 - z^2 = 4$

$x = k \rightarrow 5k^2 - y^2 - z^2 = 4$ (circle/pt/nothing)

$y = k \rightarrow 5x^2 - k^2 - z^2 = 4$ (**hyperbola**)

$z = k \rightarrow 5x^2 - y^2 - k^2 = 4$ (**hyperbola**)

Also note: $-5x^2 + y^2 + z^2 = -4$

Name: **Hyperboloid of Two Sheets**

4. $x^2 - 2y^2 + z^2 - 6 = 0$

$x = k \rightarrow k^2 - 2y^2 + z^2 - 6 = 0$ (**hyp**)

$y = k \rightarrow x^2 - 2k^2 + z^2 - 6 = 0$ (circle)

$z = k \rightarrow x^2 - 2y^2 + k^2 - 6 = 0$ (**hyp**)

Also note: $x^2 - 2y^2 + z^2 = 6$

Name: **Hyperboloid of One Sheet**

5. $-x^2 + y^2 + 4z^2 = 0$

$x = k \rightarrow -k^2 + y^2 + 4z^2 = 0$ (ellipse/pt)

$y = k \rightarrow -x^2 + k^2 + 4z^2 = 0$

(hyperbola/lines)

$z = k \rightarrow -x^2 + y^2 + 4k^2 = 0$

(hyperbola/lines)

Also note: $x^2 = y^2 + 4z^2$

Name: **Elliptical Cone**